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NEUTRON SPECTROSCOPIC FACTORS IN ^{208}Pb FROM THE PROTON DECAY OF THE ^{208}Bi ISOBARIC ANALOG STATE

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Accurate spectroscopic factors for neutron removal from ^{208}Pb, leading to single neutron hole states in ^{207}Pb, are derived from the escape widths of the isobaric analog state of the ^{208}Pb ground state in ^{208}Bi. The accuracy of the analysis, which follows a new procedure, is discussed. Derived lower limits on the occupancies of neutron orbitals in ^{208}Pb are 0.87 ± 0.06 ($p_{1/2}$), 0.92 ± 0.08 ($p_{3/2}$), 0.88 ± 0.12 ($f_{5/2}$) and a lower limit 0.7 ($f_{7/2}$).

The occupancy of single-particle orbitals below the Fermi surface has received ample interest over recent years. A depletion of up to 30%, due to ground-state correlations, has been predicted from theoretical calculations [1,2] for the double-magic nucleus ^{208}Pb. Recent (e, e'p) knockout reactions indicate a much stronger depletion, up to about 50%, for all except for the lightest targets [3–6], when derived directly in the DWIA approximation. When calibrating on the charge-density difference between the ground states of ^{206}Pb and ^{205}Tl, established from elastic electron scattering to be $\Delta\rho = z\rho(s_{1/2})$, with $z = 0.7$ [7], and using the experimental ratios of spectroscopic $s_{1/2}$ strength $\sum_f S_f(206)/\sum_f S_f(208)$ and $\sum_f S_f(205)/\sum_f S_f(206)$, one finds however an occupancy of 0.82 ± 0.09 for the $s_{1/2}$ orbital [8].

Recent (d, ^{3}He) experiments [9–11] find depletions between 5% and 30%. Spectroscopic strengths derived from hadron-induced picked reactions depend, however, sensitively on the choice of the optical potential for projectile and ejectile and even more on the geometry of the potential, adopted for the bound-state wave function of the transferred nucleon.

While the experimental efforts have recently concentrated on proton orbitals, the depletion effects must of course be similar for neutrons. Many neutron pickup reactions have been reported [12] from which spectroscopic factors have been extracted. Some of them have, however, been analyzed with an explicit normalization on sum rules.

As a contribution to the discussion on occupancies in the Pb region we analyze in this letter the escape width of the isobaric analog state (IAS). It is shown that this is an appropriate tool, alternative to knockout and pickup reactions, for obtaining accurate neutron removal spectroscopic factors. The accuracy of the method is investigated and the implications of the results are discussed.

The IAS of the ^{208}Pb ground state is found in ^{208}Bi at an excitation energy $E_x = 15.2$ MeV. The escape widths for the decay into the lowest single-neutron

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hole states ($p_{1/2}$, $p_{3/2}$, $f_{5/2}$ and $f_{7/2}$) in ^{207}Pb have been measured in several experiments [13–16]. These escape widths are related to neutron pickup spectroscopic factors by

$$\Gamma_f^{\uparrow}(j) = (N-Z)^{-1} C^2 S_f(j) \Gamma_{sp}^{\uparrow}(j, E_p, E_n) ,$$

where j labels the orbital and f the final state. E_p denotes the energy of the proton leaving the daughter nucleus behind in the state f and E_n is the corresponding neutron separation energy. The neutron excess $(N-Z)$ in the denominator refers to the parent nucleus (^{208}Pb).

The usual method of calculating the single-particle (proton) escape widths $\Gamma_{sp}^{\uparrow}(j, E_p, E_n)$ consists essentially of adjusting the depth of the potential well such that the phase shift of the partial wave j crosses 90° at the observed resonance energy E_p. $\Gamma_{sp}^{\uparrow}(j)$ may then either be equated to the energy difference between the 45° and 135° phase shifts [14] [#1] or to the width of the resonant amplitude of the partial wave j [13]. Under the assumption that $C^2S = 2j+1$, these methods have been used to calculate the escape widths of the IAS in ^{208}Bi [15,16], generally yielding agreement with experiment within 20%. Though these results seem to confirm the assumed full occupancy of the neutron shells, the recent interest in spectroscopic factors, especially in the lead region, necessitates a reexamination of the method and its accuracy.

We propose here a modification to the method of evaluating $\Gamma_{sp}^{\uparrow}(j)$ that makes it, in our opinion, more appropriate for the case of the IAS. It has been implicitly assumed in previous analyses like those quoted above [14–16] that the wave functions of the IAS and its parent are identical in the occupancies of the different shells and their vector couplings, but that a bound single-neutron wave function in the parent is to be replaced by that of an unbound proton to give the IAS. The usual replacement $|\mathrm{IAS}\rangle = T_- |\mathrm{g.s.}\rangle$ applies, however, also to the radial wave functions. This formal T_- operation is realized by for example Fermi beta decay or by charge exchange reactions, in a time that is short compared with the typical time, needed to travel a classical Bohr orbit. Therefore the space part of the IAS proton wave function, must be taken identical to that of the bound neutron that it replaces under the action of the T_- operator. This wave function is not an eigenfunction of the hamiltonian any more, but may be expanded over the new, unbound, eigenfunctions $\chi^+(E_f)$ of the final nucleus, which must be normalized to unity outgoing flux towards infinity.

For the bound neutron $3p_{1/2}$ wave function in ^{208}Pb, for example, the expansion selects out the $p_{1/2}$ amplitude from $\chi^+(E_f)$. The depth of the proton potential is adjusted such that the overlap $\langle \chi^+(p, E_f) | t_- | 3p_{1/2}(n, E_n) \rangle$ shows a maximum at $E_f = E_p(3p_{1/2}) = 11.50$ MeV, the experimentally observed centroid energy of the proton decay that leaves the nucleus in the $3p_{1/2}$ neutron hole state. By inspection of the phases of the amplitudes one finds that around this energy the phase goes through 90°, indicating a resonance in the $p_{1/2}$ partial wave. Furthermore, the corresponding radial wave function has two nodes ($3p_{1/2}$) within the nuclear radius.

The width of the resonance can therefore be found from the square of the overlap matrix element, which is a function that depends on the proton energy E_f. Thus,

$$\Gamma^{\uparrow}(3p_{1/2}, E_p, E_n) = \mathrm{FWHM}\{ | \langle \chi^+(p, E_f) | t_- | 3p_{1/2}(n, E_n) \rangle |^2 \} .$$

The widths for the other orbitals are obtained analogously.

The resulting widths depend on the parameters that determine the geometries of the potential wells chosen for the bound neutron and for the unbound proton. The associated uncertainties will be discussed hereafter.

Unlike hadron-induced transfer reactions, the (e, e'p) knockout reaction directly probes the r-dependence of the nucleon wave function. Leaving aside for the moment the strong depletion of strength resulting from a CDWIA approach [17] used for this reaction, such an analysis [6,18], carried through with a Woods–Saxon potential well for the single nucleon, provides the best possible estimate of the critical parameters r_{0p} and a_p. Switching off the nonlocality correction, which is done inside the bound state subroutine in the electron knockout code [17], a search was made on the reduced radius r_{0p} of an equivalent local Woods–Saxon potential for the best fit of the momentum distribution for proton knock-

[#1] For a survey of different methods used to evaluate single particle escape widths see ref. [15].

out, leading to final states in ^{207}Tl. This procedure was followed in order to comply with DWUCK4 [19], where the potential used in the bound state routine is local. The diffuseness was kept fixed at $a_p = 0.65$ fm, while the Thomas spin–orbit strength was taken as $\lambda = 25$ and the reduced Coulomb radius as $r_C = 1.25$ fm. Best fits to the momentum distributions of the individual transitions were found for $r_{0p} = 1.276$ ($s_{1/2}$), 1.292 ($d_{3/2}$), 1.278 ($h_{11/2}$), 1.254 ($d_{5/2}$) and 1.241 ($g_{7/2}$) fm, with errors of less than one percent. It should be noted here that spectroscopic factors, derived directly via CDWIA vary less than one percent for a one-percent change in r_{0p}. We adopt the unweighted average, $r_{0p} = 1.268$ fm, as the value used in the analysis for the unbound proton orbitals involved in the decay of the IAS.

In table 1 the sensitivities to r_{0p}, a_p, r_{0n} and a_n of the partial widths for decay into each of the hole states are given. Average values are $\Delta\Gamma^\uparrow/\Gamma^\uparrow \simeq 5\,\Delta r_{0p}/r_{0p}$ and $\Delta\Gamma^\uparrow/\Gamma^\uparrow \simeq 1.5\,\Delta a_p/a_p$ for the proton-geometry parameters. The sensitivities for the corresponding neutron parameters are about ten times smaller.

We adopt here for the neutron geometry the same parameters as for the protons: $r_{0n} = 1.268$ fm and $a_n = 0.65$ fm. Although the deduced widths are virtually independent of this particular choice it is worth noting that the RMS radii for this geometry are very close to those derived by Körner and Schiffer [20,21] and by Franey et al. [22] from sub-Coulomb neutron pickup reactions.

The accuracy attainable in the analysis of the decay of the IAS depends on that with which the geometry of the potential well, from which the unbound proton state is generated, can be pinned down. Though this potential is non-local, the availability of the bound proton wave functions in q-space, as measured in the ^{208}Pb(e, e′p)^{207}Tl reaction [20] allowed us to find an equivalent local potential. The available codes do not permit the full analysis to be carried out with the use of non-local potentials.

Table 1
Sensitivities of $\Gamma^\uparrow$ to the neutron and proton Woods–Saxon well parameters.

Shell	$\frac{(\Delta\Gamma^\uparrow/\Gamma^\uparrow)}{(\Delta r_{0p}/r_{0p})}$	$\frac{(\Delta\Gamma^\uparrow/\Gamma^\uparrow)}{(\Delta a_p/a_p)}$	$\frac{(\Delta\Gamma^\uparrow/\Gamma^\uparrow)}{(\Delta r_{0n}/r_{0n})}$	$\frac{(\Delta\Gamma^\uparrow/\Gamma^\uparrow)}{(\Delta a_n/a_n)}$
$p_{1/2}$	3.4	1.3	−0.23	−0.04
$f_{5/2}$	6.5	1.6	−0.01	−0.11
$p_{3/2}$	4.0	1.2	−0.12	−0.09
$f_{7/2}$	7.8	3.3	−0.003	−0.12

The assumption that the same equivalent local potential can be used for bound and for unbound proton states is the strongest assumption in our analysis.

As an estimate of the uncertainty in the geometry used to generate the unbound proton wave functions we take $\Delta r_{0p} = 0.02$ fm, equal to the difference by which inclusion of non-locality in the electron knock-out code [17] affects the reduced radius of the potential in the above analysis of bound proton wave functions. In addition we allow for an uncorrelated uncertainty $\Delta a_p = 0.02$ fm in calculating the escape widths. When however Δr_{0p} and Δa are correlated such as to reproduce the RMS radii of the bound proton states within one or two percent, the resulting errors on the escape widths are much smaller.

Another assumption made in our analysis, is implicit in the replacement of $|\mathrm{IAS}\rangle$ by $T_-|\mathrm{g.s.}\rangle$. More accurately one may write

$$|\mathrm{IAS}\rangle = \sqrt{1-\alpha^2}\,T_-|\mathrm{g.s.}\rangle + \alpha|T_<\rangle\,,$$

where the $T_<$ component is induced mixing with other states of lower isospin, believed to be mediated mostly by the anti analog state and the isovector monopole resonance [23]. Through this admixture the IAS can decay by neutron emission and, if energetically possible, by fission [24]. Neutron decay, not being hindered by the Coulomb barrier, may be estimated to be more than a hundred times more probable for the $T_<$ component than proton decay. The fact that the IAS still decays predominantly by proton emission shows that the amplitude of the $T_<$ component and therewith the difference between $|\mathrm{IAS}\rangle$ and $T_-|\mathrm{g.s.}\rangle$ is on the one-percent level. The replacement of $|\mathrm{IAS}\rangle$ by $T_-|\mathrm{g.s.}\rangle$ is therefore justified as long as one studies only the proton decay and it does not introduce any additional uncertainty in deduced spectroscopic factors.

In table 2 the experimental values of Gaarde et al. [13], Melzer et al. [14] and of Back et al. [16] are given for the escape widths into the lowest states of ^{207}Pb. These are the $\frac{1}{2}^-$ (ground state), $\frac{5}{2}^-$ ($E_x = 0.570$ MeV), $\frac{3}{2}^-$ ($E_x = 0.898$ MeV) and $\frac{7}{2}^-$ ($E_x = 2.339$ MeV) states. Decay into the $i_{13/2}$ hole state at 1.633 MeV has not been observed experimentally within the experimental uncertainties, which exceed

Table 2
Escape widths of the IAS in ^{208}Bi into neutron hole states in ^{207}Pb.

Shell	E_n (MeV)	E_p (MeV)	$\Gamma^{\uparrow}_{exp}$ [a] (keV)	$\Gamma^{\uparrow}_{exp}$ [b] (keV)	$\Gamma^{\uparrow}_{exp}$ [c] (keV)	$\Gamma^{\uparrow}_{exp}$ (av.) [d] (keV)	$\Gamma^{\uparrow}_{calc.}$ [e] (keV)	C^2S
$p_{1/2}$	−7.37	11.50	51 ±6	56.4±6.4	51.6±1.7	51.9±1.6	59.6±4.0	1.74±0.13
$f_{5/2}$	−7.94	10.93	26 ±6	25.2±2.5	29.5±4.0	26.4±2.0	30.0±3.4	5.28±0.73
$p_{3/2}$	−8.27	10.60	61 ±8	63.2±6.3	66.8±4.6	64.7±3.4	70.0±5.1	3.70±0.33
$f_{7/2}$	−9.71	9.16	3.3±0.5	5.0±0.5		4.2±0.6	8.4±1.3	4.00±0.84

[a] Ref. [13], [b] Ref. [14], [c] Ref. [16].
[d] Weighted average of refs. [13,14,16].
[e] Calculated with $r_C=1.25$ fm, $\lambda=25$, $r_0=1.268$ fm and $a=0.65$ fm for $C^2S=(2j+1)$. The errors reflect uncorrelated variations of $\Delta r_{0p}=0.02$ fm and $\Delta a_p=0.02$ fm.

the value of 0.3 keV, predicted for its width in case of full occupancy.

The experimental escape widths of refs. [13,14,16] are in good agreement with each other. It is noteworthy that they have been obtained with very different techniques: in ref. [13] the IAS is excited via the ^{208}Pb(^{3}He, t) reaction and its decay probabilities into the different hole states are measured, whereas refs. [14,16] study the resonant ^{207}Pb(p, p′)^{207}Pb reaction.

Table 2 gives the predicted escape widths associated with full occupancy of each orbital and under the assumption that the final state exhausts the full sum rule. The spectroscopic factors are obtained as

$$C^2S=(2j+1)\Gamma^{\uparrow}_{exp}(\text{av.})/\Gamma^{\uparrow}_{calc}\,.$$

For the $p_{1/2}$, $p_{3/2}$ and $f_{5/2}$ shells no spectroscopic strength has been identified on other states in ^{207}Pb [12,25] and hence lower limits for their occupancies are $v^2=0.87\pm0.06$ ($p_{1/2}$), 0.92 ± 0.08 ($p_{3/2}$) and 0.88 ± 0.12 ($f_{5/2}$).

For the $f_{7/2}$ shell, strength has been located on higher excited $\frac{7}{2}^-$ states with an estimated summed spectroscopic strength up to 30% of that on the $E_x=2.339$ MeV level [26,27]. Also the strength of the latter, relative to that of the 0.575 MeV ($f_{5/2}$) state, as observed in hadron-induced pickup reactions is lower by 30–40% [12,25]. Taking into account this fragmentation of the $f_{7/2}$ strength, the ratio of the experimental and calculated escape widths has to be increased by probably at least 40% in order to arrive at an estimate for the occupancy of the $f_{7/2}$ shell in ^{208}Pb, giving $v^2>0.70$.

In conclusion, we have shown the potential of the escape widths of the IAS as a means of arriving at accurate neutron removal spectroscopic factors.

The method is virtually insensitive to the choice of the geometry of the potential well of the bound neutron, in contrast to hadron-induced pickup reactions. As an example, a ^{208}Pb(d, t)^{207}Pb reaction at 50 MeV incident energy, leading to the same final states as studied here, has typical sensitivities $\Delta C^2S/C^2S\simeq -10\,\Delta r_{0n}/r_{0n}$ and $\simeq -1.2\,\Delta a_n/a_n$. In addition there is an uncertainty of about 20% associated with the choice of optical model parameters. The uncertainties on the single particle escape widths of the IAS and therewith on the spectroscopic factors associated with the geometry of the potential are smaller (table 1) than for this transfer reaction and optical model parameters do not enter in the analysis. Occupancies for orbitals closest to the Fermi surface, when identified with the lower limits, set by the spectroscopic factors on the hole states under study, appear to be depleted to values not in disagreement with theoretical predictions ($v^2=0.7$–0.9) [1,2]. Recent calculations by Mahaux and Sartor [28] and by Johnson et al. [29], in which the mean field is given a dispersive term in addition to the Hartree–Fock type mean field, yield average occupancies of 0.79 and 0.82, respectively, for the neutron 3p and 2f shells.

The average occupation observed for the $3p_{1/2}$, $3p_{3/2}$ and $2f_{5/2}$ valence neutron orbitals, 0.89 ± 0.05, agrees well with the experimental value 0.82 ± 0.09 derived for the $3s_{1/2}$ proton occupation in ^{208}Pb from the relative (e, e′p) measurements, but not with the much smaller occupancies for proton orbitals as derived from a direct CDWIA analysis of recent (e, e′p) data [18].

Finally the spectroscopic factor found in this work for $3p_{1/2}$ neutron pickup may be used to estimate the

contribution of this orbital to the static magnetic moment ($q=0$) of the ^{207}Pb ground state. Experimentally this magnetic moment is found to be reduced by a factor 0.908 [30] compared with the Schmidt value. The contribution of the $3p_{1/2}$ shell to the static magnetic moment may be estimated by use of the spin dependent sum rule as

$$\mu_0(3p_{1/2}) = -\tfrac{1}{6}g_n \sum G(3p_{1/2}, I_f^\pi = 0^+) + \tfrac{1}{18}g_n \sum G(3p_{1/2}, I_f^\pi = 1^+)$$

by summing over spectroscopic strengths for neutron stripping to states in ^{208}Pb. No $3p_{1/2}$ strength other than to the ground state has been reported [31]. Restricting the summation to the ground state and using the value of the spectroscopic strength found in the present work one estimates a contribution of 0.87 ± 0.06 of the Schmidt value ($-\frac{1}{6}g_n$). Additional strength on 0^+ states would bring this value closer to the Schmidt value and strength on 1^+ states, if at all present, has little weight in the above expression. Thus it seems that the contribution of the $3p_{1/2}$ orbital essentially accounts for the observed static moment when the spectroscopic factor derived in this work is used. A reduction of the magnetic form factor at high momentum transfer ($q = 1.3$–$2.6\ \mathrm{fm}^{-1}$) to about half the single $3p_{1/2}$ hole estimate has been reported by Papanicolas et al. [32]. This quenching cannot be explained by a reduction of the $3p_{1/2}$ contribution as we have shown above and is at present not understood.

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